Naturalness, dark matter(s) and a last bet for a Susy Higgs.

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with Graham Ross and Sebastian Cassel

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• Standard Model (SM): best model so far.

- Agrees with experiment. Open questions: hierarchy, EWSB, unification, DM, gravity,....

- $\delta m_h^2 \sim \mathcal{F}(\alpha_i^0, m_j^0)\,\Lambda_{UV}^2, \quad \Lambda_{UV} \sim M_{Planck}$: $\Rightarrow$ Hierarchy problem $\Leftrightarrow$ fine tuning (naturalness).

- two faces of the same problem.

• MSSM. SUSY searches.

- $\delta m_h^2 \sim m_{susy}^2 \log \Lambda_{UV}^2/m_{susy}^2, \quad m_{susy} \sim$ TeV; $\quad m_{susy} \sim \Lambda_{UV} \Rightarrow$ back to SM fine-tuning

$$\mathcal{L} = \int d^4\theta \sum_{\Phi} \mathcal{Z}_\Phi\, \Phi^\dagger e^V \Phi + \Big\{ \sum_{i=1,2,3.} \int d^2\theta\, \frac{1}{g_i^2}\, (1 + m_{1/2}\theta\theta)\ \mathrm{Tr}\, W^\alpha W_\alpha\vert_i + h.c. \Big\}$$

$$+ \int d^2\theta \Big[H_2\, Q\, \lambda_U\, U^c + Q\, \lambda_D\, D^c\, H_1 + L\, \lambda_E\, E^c\, H_1 + \mu\, H_1\, H_2 \Big] + h.c., \quad \Phi : H_{1,2}, Q, U^c, D^c, E^c, L$$

$$\mathcal{Z}_\Phi(S, S^\dagger) = 1 - S\, S^\dagger,\ S = \theta\theta\, m_0;\ \ m_0 = \frac{\langle F_{hid}\rangle}{M_P},\ \ \lambda_F(S) = \lambda_F(0)(1 + A_0\, S),\ \ \mu(S) = \mu_0\,(1 + B_0 S)$$

$$A_0, B_0, \mu_0, m_0, m_{1/2}, \tan\beta = v_2/v_1$$

- MSSM scalar potential:

$$
\begin{aligned}
V \;&=\; m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (B_0\,\mu_0\,H_1\cdot H_2 + h.c.) + \lambda_1/2\,|H_1|^4 + \lambda_2/2\,|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2 \\
&+\; \lambda_4\,|H_1\cdot H_2|^2 + \Big[\lambda_5/2\,(H_1\cdot H_2)^2 + \lambda_6\,|H_1|^2\,(H_1\cdot H_2) + \lambda_7\,|H_2|^2\,(H_1\cdot H_2) + h.c.\Big]
\end{aligned}
$$

$$
\text{Tree-level: } \lambda_{1,2} = (g_1^2+g_2^2)/4, \quad \lambda_3 = (g_2^2-g_1^2)/4, \quad \lambda_4 = -g_2^2/2, \quad \lambda_{5,6,7} = 0
$$

$$
m^2 \;\equiv\; m_1^2\,\cos^2\beta + m_2^2\,\sin^2\beta - B_0\,\mu_0\,\sin 2\beta, \quad \mathrm{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2
$$

$$
\lambda \;\equiv\; \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\,\big(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\big)
$$

- The Problem:

$$
v^2 = -m^2/\lambda, \qquad v = \mathcal{O}(100\ \mathrm{GeV}), \quad \lambda < 1, \qquad \text{but} \quad m,\, m_{1,2}, B_0 \sim \mathcal{O}(1\,\mathrm{TeV}).
$$

- "residual" fine-tuning (little hierarchy). Tree level: $m_h < m_Z$, LEP2: > 114.4 GeV
- Need: large quantum corrections (QC) $\Rightarrow$ large m_{susy}. But QC can also increase λ and $m_h^2 \propto \lambda\, v^2$
- This is not only a problem of scales but of couplings (λ_{MSSM} small). $\delta\lambda > 0$ vs. $m_{susy}^2 \sim m^2$

• EW fine-tuning measures as a test of SUSY [Ellis et al 1986, Barbieri 1988]

$$\Delta \equiv \max \big|\Delta_p\big|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \qquad \text{or} \quad \Delta' \equiv \Big(\sum_p \Delta_p^2\Big)^{1/2}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p}$$

$$v^2 = -m^2/\lambda, \quad 2\lambda \frac{\partial m^2}{\partial \beta} = m^2 \frac{\partial \lambda}{\partial \beta}, \quad \Rightarrow \quad m^2, \lambda = F(p, \beta(p)),$$

• Exact formula:

$$\Delta_p = -\frac{p}{z}\left[\left(2\frac{\partial^2 m^2}{\partial \beta^2} + v^2 \frac{\partial^2 \lambda}{\partial \beta^2}\right)\left(\frac{\partial \lambda}{\partial p} + \frac{1}{v^2}\frac{\partial m^2}{\partial p}\right) + \frac{\partial m^2}{\partial \beta}\frac{\partial^2 \lambda}{\partial \beta \partial p} - \frac{\partial \lambda}{\partial \beta}\frac{\partial^2 m^2}{\partial \beta \partial p}\right]$$

$$z \equiv \lambda\left(2\frac{\partial^2 m^2}{\partial \beta^2} + v^2\frac{\partial^2 \lambda}{\partial \beta^2}\right) - \frac{v^2}{2}\left(\frac{\partial \lambda}{\partial \beta}\right)^2, \qquad \Delta_p = \frac{p}{m_Z^2(1+\delta)} + \mathcal{O}\Big[\frac{1}{\tan\beta}\Big], \;\; \delta : \text{QC top Y.}$$

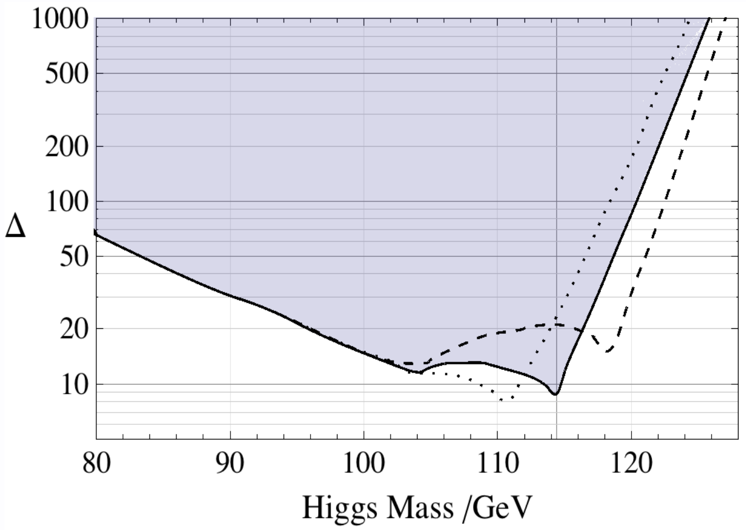
• $p_i \sim 1/\Delta$: "effective prior" emerges automatically in Bayesian fits [Casas et al 2008]

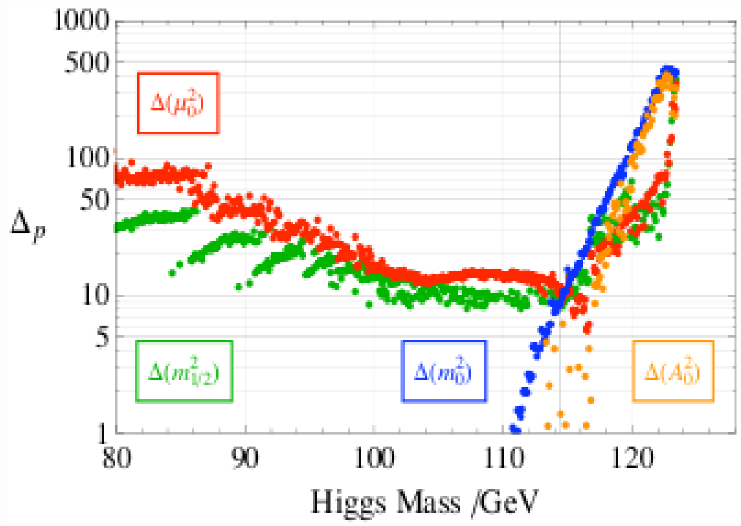
• if Δ too large: SUSY fails... Wanted: Δ minimal, for low UV sensitivity.

• Many studies: Δ: 1-loop. $\Delta \sim 100$ but $\Delta \sim m_0^2 \sim \exp(m_h^2/v^2)\,!$. [Pokorski, Ellis et al]

• 2-loop study (SoftSusy, micrOMEGAs: 6years 30×3 GHz). Impose usual constraints: TH + EXP but NO LEP bound on m_h!

• 2-loop results in CMSSM: min $\Delta \Rightarrow \; m_h = ?$ [S. Cassel, DG, G. Ross]





$(\alpha_3, m_t) = (0.1176, 173.1)$.

$\Delta \equiv \max|\Delta_p|, \quad p = \mu_0^2, m_0^2, A_0^2, B_0^2, m_{1/2}^2,$

$\Delta_{\mu_0^2}$, $\Delta_{m_0^2}$ dominant (EW vs. QCD)

$\Rightarrow$ minimal $\Delta \propto \exp(m_h^2/v^2) \Rightarrow$ 2-loop effects important!

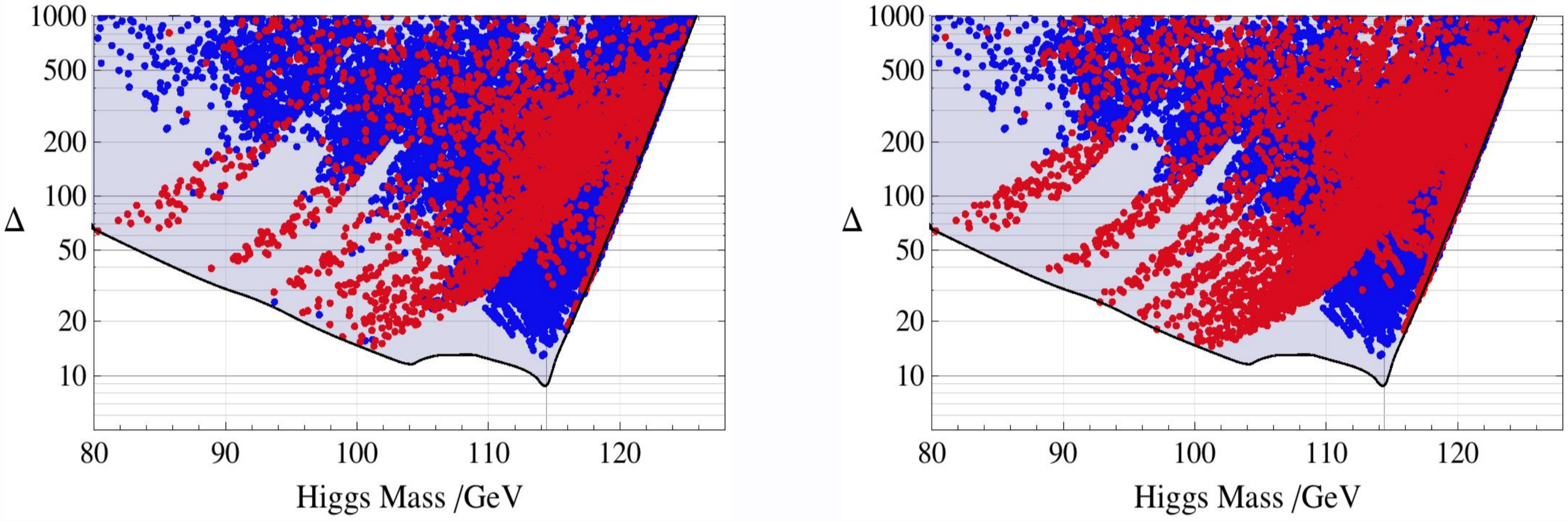
$\Rightarrow$ without LEP2 bound on m_h, min Δ: $\Delta \approx 9 \;\Rightarrow\; m_h = 114 \pm 3$ GeV, (just above LEP2 bound!)

$\Rightarrow$ CMSSM: Δ less than usually thought, when using the correct, 2-loop Δ.

$\Rightarrow$ Same result (m_h) and behaviour if using min of $\Delta' = \sqrt{\sum_p \Delta_p^2}$, (1 to 2 GeV variations).

$\Rightarrow$ dotted line: increase α_3 (1σ), reduce m_t (1σ) $\Rightarrow$ QCD does not like large m_h! (fine-tuning cost)

• 2-loop results in CMSSM: min Δ + dark matter $\Rightarrow$ $m_h = ?$ [S. Cassel, DG, G. Ross]



LSP: good dark matter candidate $\Rightarrow$ dark matter constraint:

WMAP: $\Omega h^2 = 0.1099 \pm 0.0062$; blue: Ωh^2 not-saturated; red: Ω saturated: 1σ (left); 3σ (right).

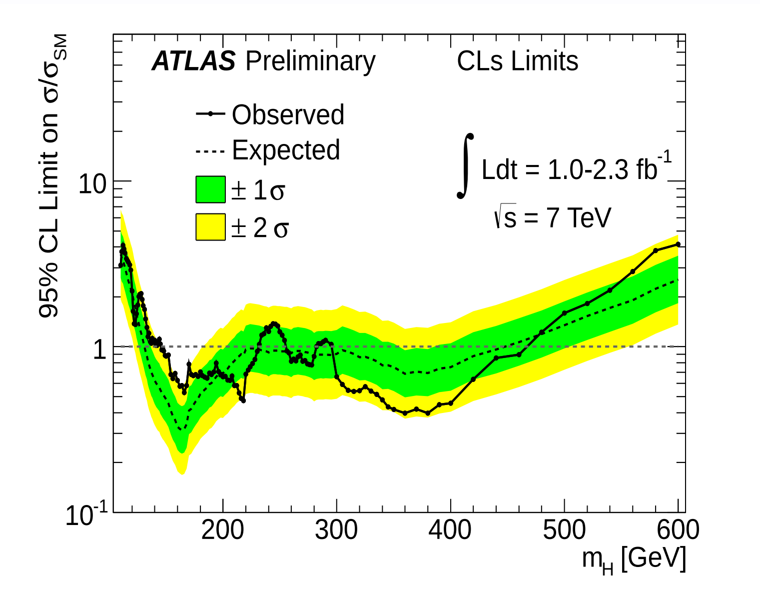
$\Rightarrow$ Min Δ + "right" dark matter abundance, (no LEP2 bound): (similar Δ')

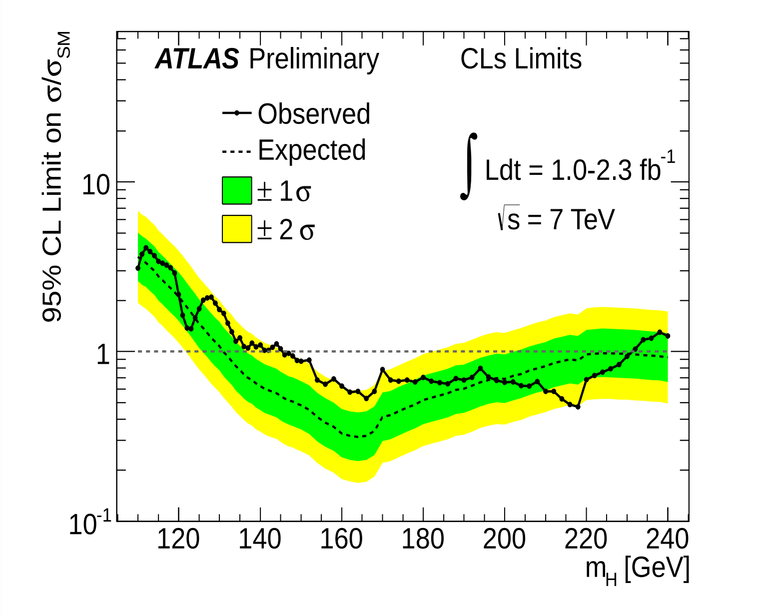
$$m_h = 114.7 \pm 3 \text{ GeV}, \quad \Delta = 15.0, \quad \text{(consistent with WMAP bound).}$$
$$m_h = 115.9 \pm 3 \text{ GeV}, \quad \Delta = 17.8, \quad \text{(saturating WMAP within } 3\sigma\text{).}$$

• an upper bound on $\Delta \Rightarrow$ bounds on SUSY spectrum. $\Delta < 1000 \Rightarrow m_h < 126$ GeV ($\pm$ 3 GeV)

$\Delta < 100 \Rightarrow m_h < 121$ GeV ($\pm$ 3 GeV)

- What recent data says....current exclusion plots for a SM-like Higgs ($1/fb$):



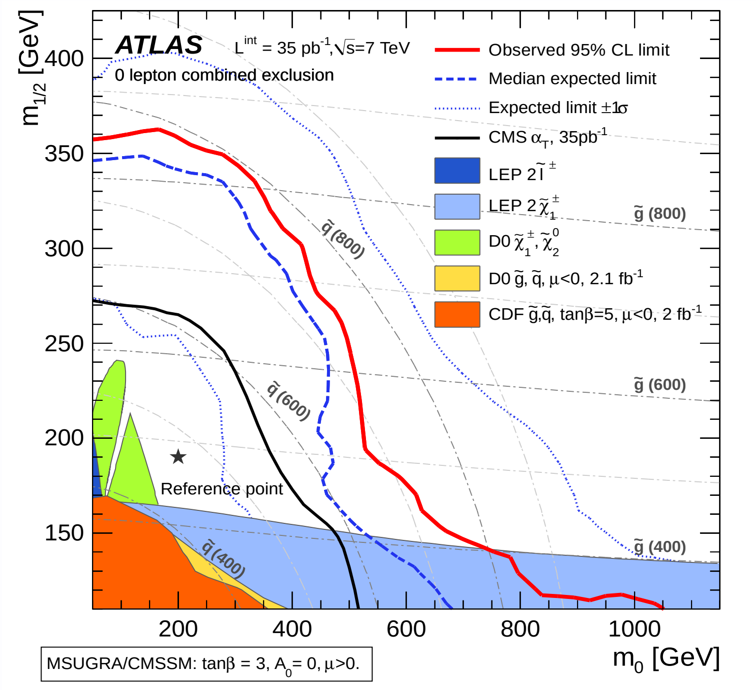


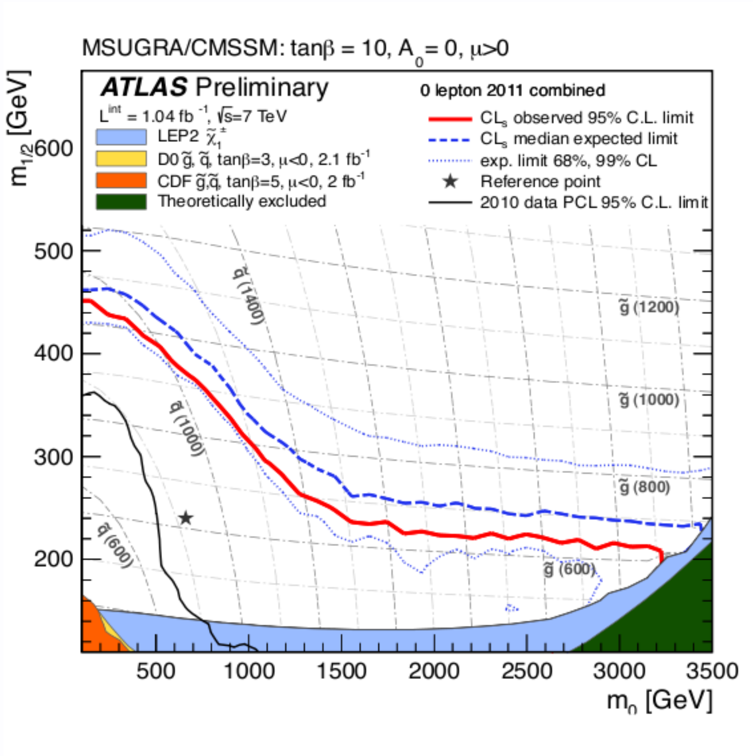
⇒ The combined upper limit on the Standard Model Higgs boson production cross section divided by the Standard Model expectation, as a function of m_h – the solid line.

⇒ The dotted line shows the median expected limit.

⇒ m_h range left consistent with Susy (< 145 GeV).

- CMSSM and recent Atlas SUSY searches:



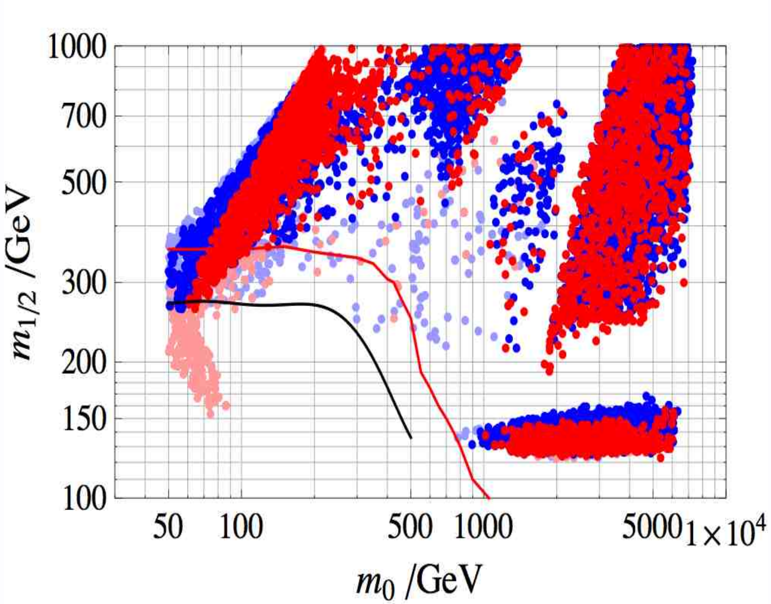


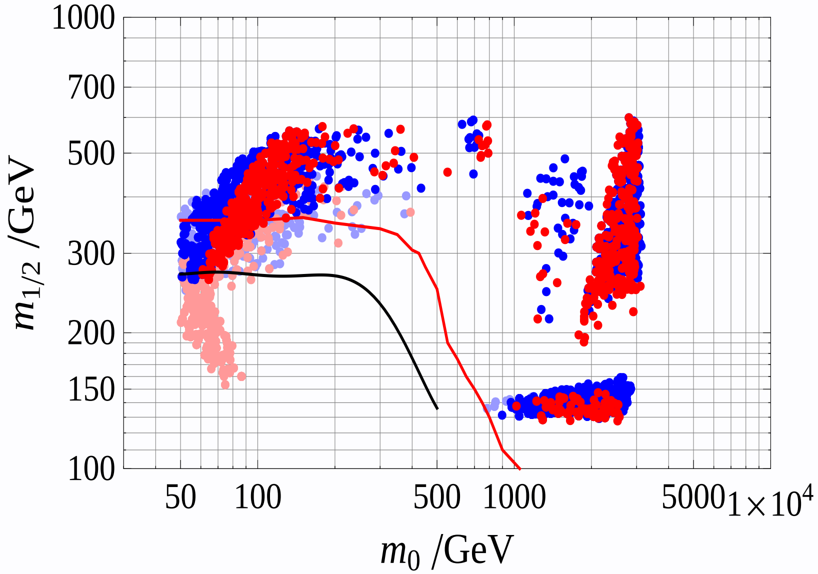
$\Rightarrow$ Left: Atlas exclusion limits (red curve, observed) for $\tan\beta = 3$, $A_0 = 0$, $\mu > 0$. arXiv:1102.5290.

$\Rightarrow$ Right: Atlas, for $\tan\beta = 10$, at 1.04/fb. Aug 2011.

Atlas is already ruling out large regions of the CMSSM .....

• 2-loop results in CMSSM: min Δ & the LHC @ 7 TeV reach. [S. Cassel, DG, G. Ross]





$\Rightarrow$ Left: points $\Delta < 1000$: blue: consistent with WMAP (3σ deviation); red: saturate it within 3σ.

Points in lighter (darker) red/blue have m_h below (above) LEP2 bound for m_h.

$\Rightarrow$ Right: points $\Delta < 100$:

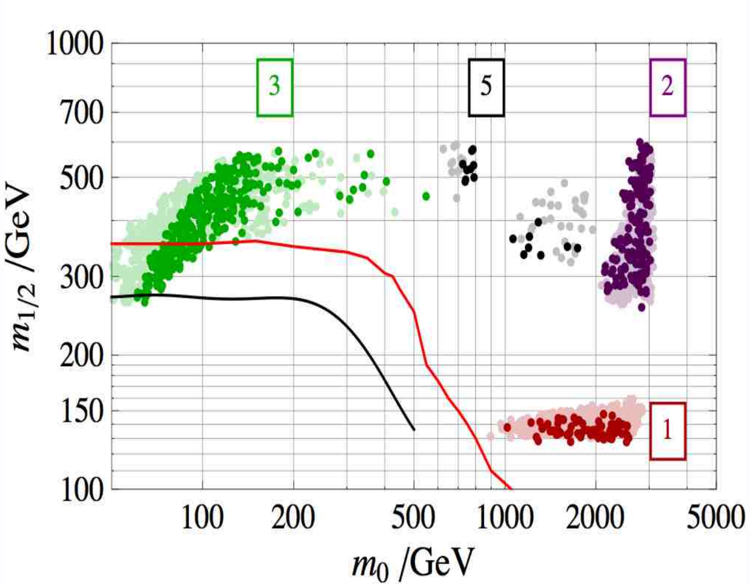
$m_h < 121$ GeV, $m_0 < 3200$, $120 < m_{1/2} < 720$, $\mu < 680$, $-2000 < A_0 < 2500$, GeV, $\tan\beta > 5.5$

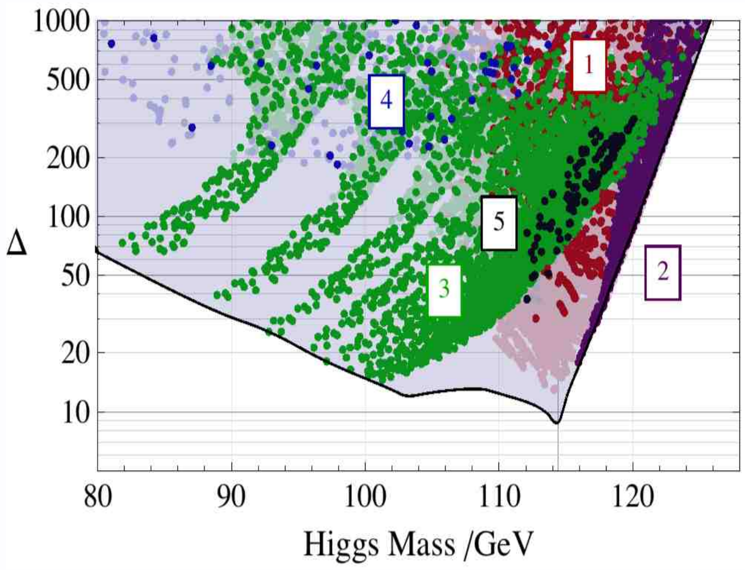
$\chi_1^0 < 305$, $\chi_2^0 < 550$, $\chi_3^0 < 660$, $\chi_4^0 < 665$, $\chi_1^\pm < 550$, $\chi_2^\pm < 670$, $\tilde{g} < 1700$, $\tilde{t}_1 < 2080$, $\tilde{t}_2 < 2660$

$\tilde{b}_1 < 2660$, $\tilde{b}_2 < 3140$ GeV

$\Rightarrow$ small area in $(m_0, m_{1/2})$: not a measure of (large) fine-tuning! $\Rightarrow$ "moduli fixing" $\neq$ fine tuning

• 2-loop results in CMSSM: min Δ and LHC @ 7 TeV reach; $m_h > 114$ GeV.





$\Delta < 100$; $m_h > 111$ GeV, CDMS-II. dark colours: Ωh^2 within 3σ; light colours: below this bound.

- purple: LSP: higgsino 10%, heavy squarks
- red: LSP: bino-like, heavy squarks (TeV).
- green: LSP: bino-like. light squarks. [S. Cassel, DG, S. Kraml, G. Ross, A. Lessa]

- purple points preferred under current exp constraints (CMSSM): min $\Delta = 18$: $m_h = 115.9 \pm 3$ GeV

• How do these results m_h change under "new physics" ? - large(r) m_h at low Δ ?

(a). to relax CMSSM constraints (gaugino univ); symmetries... [Kane, King, Ross, Horton, 09]

(b). "new physics" beyond MSSM, that increases m_h and reduces Δ: $U(1)'$, S, D of SU(2)...

Effective approach: MSSM Higgs+ effective operators of d=5, d=6.

$$\mathcal{L}_1 = \frac{1}{M_*}\int d^2\theta\ \zeta(S)\,(H_2.H_1)^2 + h.c., \qquad [S,T]$$

$$\mathcal{L}_2 = \frac{1}{M_*}\int d^4\theta\ \Big\{ a(S,S^\dagger) D^\alpha \Big[b(S,S^\dagger)\, H_2\, e^{-V_1}\Big] D_\alpha \Big[c(S,S^\dagger)\, e^{V_1}\, H_1\Big] + h.c.\Big\}, \quad [D]$$

- generated in renormalisable theories, after integrating some massive states.

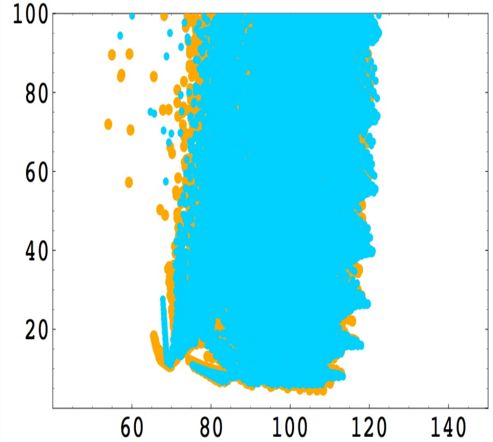
• Consider: (d=5), (d=6) operators; corrections to m_h?

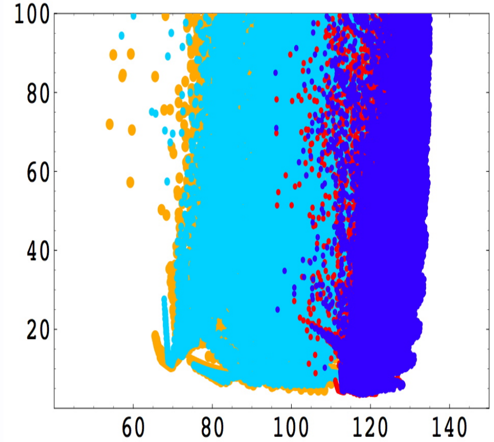
$$\mathcal{L}_{MSSM}^{Higgs} + \mathcal{L}_1(\propto 1/M_*) + \mathcal{O}_i(\propto 1/M_*^2)$$

• $\mathcal{O}_i$: d=6 operators ($U(1)$, S, D,...): [DG, I. Antoniadis, E. Dudas]

• Fine-tuning: MSSM + (d=5) operator: $\mathcal{L}_1 = \int d^2\theta\,(\zeta_0 + \zeta_1 S)(H_1.H_2)^2$ [Cassel, DG, Ross]

Δ





MSSM m_h MSSM+$\mathcal{L}_1$, $\mu_0\zeta_0 = 0.035$, $\zeta_1 = 0$

$$m^2_{h,H} = \left(m_h^2\right)^{1-loop}_{MSSM} + 2\zeta_0\,\mu_0\;v^2\sin 2\beta\left(1 \pm \frac{m_A^2 + m_Z^2}{\sqrt{w}}\right) + \zeta_1\,m_0 v^2\left(1 \mp \frac{(m_A^2 - m_Z^2)\cos^2 2\beta}{\sqrt{w}}\right) + \delta m_h^2$$

where $\zeta_0,\,\zeta_1 \sim \mathcal{O}(1/M_*)$, $\delta m_h^2 = \mathcal{O}(1/M_*^2)$, $w \equiv [m_A^2 + m_Z^2]^2 - 4m_A^2 m_Z^2\cos^2 2\beta$

$\Rightarrow$ $\Delta < 10$, $114.4 \leq m_h \leq 130$ GeV, $M_* \approx 1/\zeta_0 \approx 65 \times \mu_0 = 5$ to 10 TeV, $\tan\beta < 6$.

$\Rightarrow$ (d=5) op: massive singlet: $S\,H_1\,H_2 + M_* S^2$. $\Rightarrow$ NMSSM with $M_* S^2$ F-term.

$\Rightarrow$ At large $\tan\beta$: d=6 operators relevant: $\lambda \propto (2\mu_0\,\zeta_0)^2 \sim (2\,\zeta_0\mu_0)/\tan\beta \Rightarrow (2\mu_0\zeta_0) < 1/\tan\beta$

- Conclusions:

- Hierarchy problem $\leftrightarrow$ fine tuning. Test of SUSY.

$\Rightarrow$ min Δ + DM consistency, in constrained MSSM, no LEP2 bound on m_h:

$$\begin{aligned} m_h &= 114 \pm 3 \quad \text{GeV}, \quad \Delta \approx 9, \quad \text{(no DM constraint). (?)} \\ m_h &= 115.9 \pm 3 \ \text{GeV}, \quad \Delta = 18, \quad \text{(WMAP within } 3\sigma\text{)}. \end{aligned}$$

$\Rightarrow$ Δ min near LEP2 bound! similar result Δ' in quadrature [1-2 GeV difference, $m_h > 120$ GeV]

$\Rightarrow$ QCD does not like large m_h without fine-tuning cost: $\Delta < 100$ (1000), $m_h < 121$ (126) GeV.

$\Rightarrow$ "dark matters": g-2 discrepancy: v. strong impact on CMSSM with $\Delta < 100$....

- Beyond MSSM Higgs with effective operators of d=5, d=6 (new U(1)'s, S, D...):

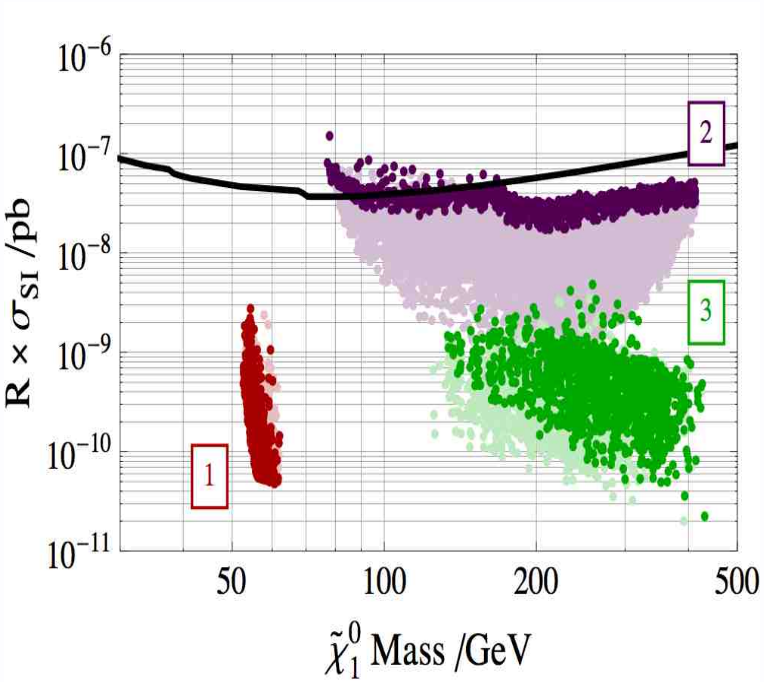
$\Rightarrow$ d=5 operators: small $\Delta < 10$ allowed for $114.4 \leq m_h \leq 130$ GeV. Massive S? NMSSM with $M_* S^2$

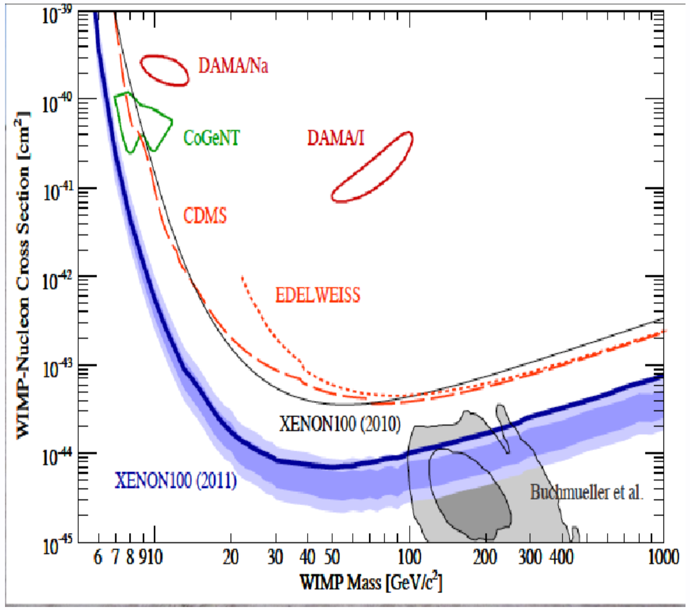
$\Rightarrow$ d=6 operators:

Points of $\Delta < 100$ (200) $\Rightarrow$ SUSY $\delta m_h \leq 4$ GeV (6 GeV). $M_* = 8$ TeV; (± 1 GeV for ∓ 1 TeV)

Extra U(1) or S ?

• Impact of DM experiments on low-fine tuned regions.





⇒ Left: Impact of CDMS (black line) on low fine-tuning points.

⇒ Right: wimp-nucleon cross section function of wimp mass (Aprile et al 1104.2549)

- d=6 operators beyond MSSM Higgs: [Antoniadis, Dudas, DG, Tziveloglou]

$$
\begin{aligned}
\mathcal{O}_i &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_i(S,S^\dagger)\ (H_i^\dagger\, e^{V_i}\, H_i)^2, \qquad i=1,2. \qquad (T,U(1))\\
\mathcal{O}_3 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_3(S,S^\dagger)\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2), \qquad (T,U(1))\\
\mathcal{O}_4 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_4(S,S^\dagger)\ (H_2\, H_1)\,(H_2\, H_1)^\dagger, \qquad (S)\\
\mathcal{O}_5 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_5(S,S^\dagger)\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2\, H_1+h.c.), \qquad (2\,D,S)\\
\mathcal{O}_6 &= \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_6(S,S^\dagger)\ (H_2^\dagger\, e^{V_2}\, H_2)\,(H_2\, H_1+h.c.), \qquad (2\,D,S)\\
\mathcal{O}_7 &= \frac{1}{M_*^2}\int d^2\theta\ \frac{1}{16\kappa\, g_i^2}\mathcal{Z}_7(S,0)\ \mathrm{Tr}\, W_i^\alpha\, W_{i,\alpha}\,(H_2\, H_1)+h.c.,
\end{aligned}
$$

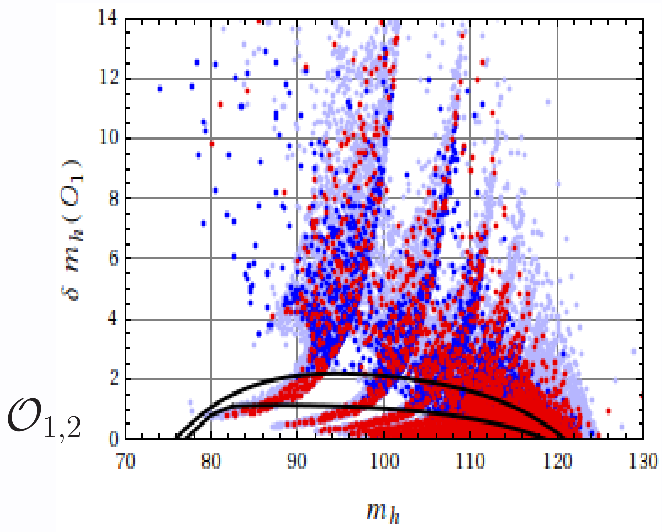
also 8 derivative operators, examples:

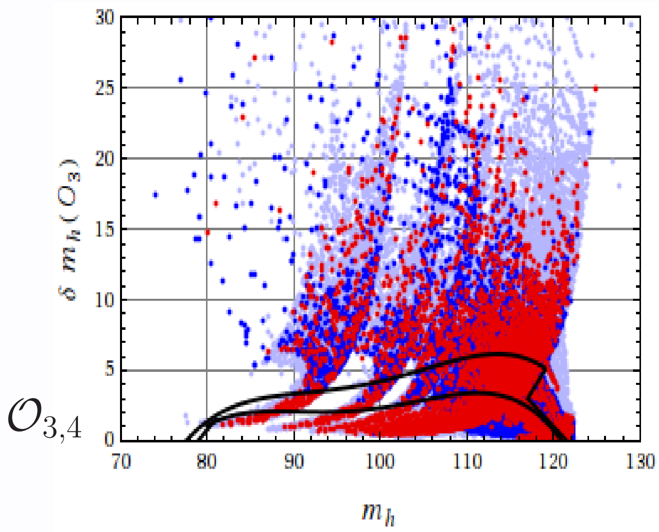
$$
\mathcal{O}_8 = \frac{1}{M_*^2}\int d^4\theta\ \mathcal{Z}_9(S,S^\dagger)\ H_1^\dagger\,\overline{\nabla}^2\, e^{V_1}\,\nabla^2\, H_1, ...etc.......
$$

$$
\frac{1}{M_*^2}\,\mathcal{Z}_j(S,S^\dagger) = \alpha_{j0}+\alpha_{j1}\, m_0\,\theta\theta+\alpha_{j1}^*\, m_0\,\overline{\theta\theta}+\alpha_{j2}\, m_0^2\,\theta\theta\overline{\theta\theta}, \qquad \alpha_{jk}\sim 1/M_*^2, \quad S=m_0\,\theta\theta
$$

$\Rightarrow \mathcal{O}_{8,....,15}$ removed by field redefinitions up to ren of wavefunction, soft terms, μ! $\mathcal{L}\to\mathcal{L}+\mathcal{O}_{1,....,7}$.

- d=6 effective operators: corrections to m_h $(\alpha_{jk} \sim 1/M_*^2)$





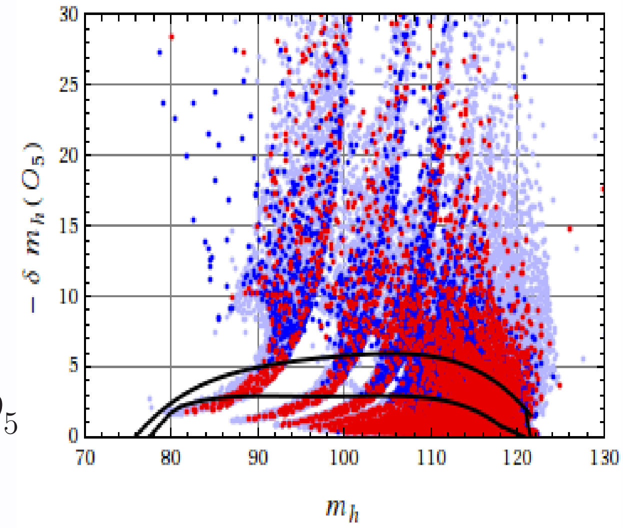
$$\delta\, m_h^2 \;=\; -2\, v^2 \left[\alpha_{22}\, m_0^2 + (\alpha_{30}+\alpha_{40})\mu_0^2 + 2\alpha_{61}\, m_0\, \mu_0 - \alpha_{20}\, m_Z^2\right] - \frac{(2\,\zeta_0\,\mu_0)^2\;v^4}{m_A^2 - m_Z^2}$$
$$+\frac{v^2}{\tan\beta}\left[\frac{1}{(m_A^2-m_Z^2)}\Big(4\, m_A^2\,\big(\,(2\alpha_{21}+\alpha_{31}+\alpha_{41}+2\alpha_{81})\, m_0\,\mu_0 + (2\alpha_{50}+\alpha_{60})\,\mu_0^2 + \alpha_{62}\, m_0^2\big)\right.$$
$$\left. -(2\alpha_{60}-3\alpha_{70})\, m_A^2\, m_Z^2 - (2\alpha_{60}+\alpha_{70})\, m_Z^4\Big) + \frac{8\,(m_A^2+m_Z^2)\;(\mu_0\, m_0\,\zeta_0\,\zeta_1)\, v^2}{(m_A^2-m_Z^2)^2}\right] + \mathcal{O}\left[\frac{\tilde m^2}{M_*^2\tan^2\beta}\right]$$

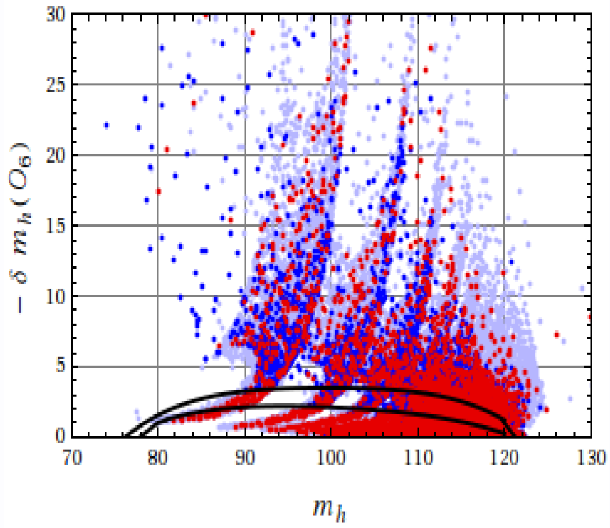
$$\Rightarrow\quad \delta m_h = \big(m_h^2+\delta m_h^2\big)^{1/2} - m_h = (1/2)\;(\delta m_h^2/m_h) + \mathcal{O}(1/M_*^4) \qquad m_h:\ \text{MSSM 2-loop LL value}$$

$\Rightarrow$ lower curve: $\Delta < 100$: $m_h < 121$ GeV, $\delta m_h < 4$ GeV. $M_* = 8$ TeV. note: $\alpha_{j0}\tilde m \leq 1/4$

$\Rightarrow$ top curve: $\Delta < 200$: $m_h < 122$ GeV, $\delta m_h < 6$ GeV. $\pm 1\,\mathrm{GeV}\;(\delta m_h) \leftrightarrow \mp 1$ TeV (δM_*).

• d=6 effective operators: corrections to m_h, $(\alpha_{jk} \sim 1/M_*^2)$

$\mathcal{O}_5$ 

$\mathcal{O}_6$ 

$\rho - 1 = -(v^2/M_*^2)\left[\alpha_{10}\cos^4\beta + \alpha_{20}\sin^4\beta - \alpha_{30}\sin^2\beta\cos^2\beta\right] + \mathcal{O}(v^4/M_*^4)$, $M_* \sim 8$ TeV [Blum et al]

$\Rightarrow$ large $\tan\beta$: larger α_{30}, α_{10} allowed $\Rightarrow$ α_{30}, α_{40} largest SUSY correction to m_h

$$\mathcal{O}_3 \sim \alpha_{30} \int d^4\theta \;\; (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2), \;\; [T, U(1)] \quad \mathcal{O}_4 \sim \alpha_{40} \int d^4\theta \;\; (H_2\, H_1)\,(H_2\, H_1)^\dagger, \;\; [S]$$

$\Rightarrow$ difficult to generate α_{30}, α_{40} with the "right" sign, by integrating massive $T, U(1), S$ in ren model

$\Rightarrow$ neutralino mass corrections very small (LSP): $\sim \mu/M_* \sim$ few (≤ 1 to 2 GeV)!

$\Rightarrow$ $\Delta = 18$ (3σ WMAP), $m_h = 115.9 \pm 2$ GeV $\Rightarrow$ $m_h + \delta m_h = 119.9 \pm 2$ GeV, $\Delta' = \Delta \frac{m_h^2}{(m_h + \delta m_h)^2}$

- Generating d=5 operators beyond the MSSM the Higgs sector:

- $\mathcal{L}_1$?: massive gauge singlet Σ or SU(2) triplet ($M_* \gg \mu$):

$$\mathcal{L} \supset \int d^4\theta\, \Sigma^\dagger \Sigma + \int d^2\theta\, \Big[\mu\, H_1.H_2 - M_*\, \Sigma^2 + \lambda\, \Sigma\, H_1\, H_2\Big] + h.c. \;\; \Rightarrow \;\; \mathcal{L}_1 = \frac{1}{4M_*}\int d^2\theta\; \lambda^2\; (H_1.H_2)^2 + h.c.$$

- $\mathcal{L}_2$?: massive Higgs doublets $H_{3,4}$ beyond MSSM $H_{1,2}$

$$\mathcal{L} \;=\; \int d^4\theta \, \Big[\sum_{j=1,3} \big(H_j^\dagger\; e^{V_1}\, H_j + H_{j+1}^\dagger\; e^{V_2}\, H_{j+1}\big) + \big(\nu_1 H_1^\dagger\; e^{V_1}\, H_3 + \nu_2\, H_2^\dagger\, e^{V_2}\, H_4 + h.c.\big)\Big];$$

$$+ \int d^2\theta\, \big[\mu\, H_1\, H_2 + M_*\, H_3\, H_4\big] + h.c.; \qquad -\frac{1}{4}\overline{D}^2\big[H_3^\dagger\, e^{V_1}\big] - \frac{1}{4}\overline{D}^2\big[\nu_1\, H_1^\dagger\, e^{V_1}\big] + M_*\, H_4 = 0\; (H_3)$$

$$\Rightarrow\; \mathcal{L}_2 = \int d^4\theta \Big[\sum_{j=1,2} H_j^\dagger\; e^{V_j}\, H_j + \Big(\frac{\nu_1\, \nu_2}{4\, M_*}\; H_2\, e^{-V_1}\, D^2\, e^{V_1}\, H_1 + h.c.\Big)\Big] + \Big[\int d^2\theta\;\; \mu\, H_1.\, H_2 + h.c.\Big]$$

$$H_2\, e^{-V_1}\, D^2\, e^{V_1}\, H_1 \;\sim\; D^\alpha\, \big[H_2\, e^{-V_1}\big]\, D_\alpha\, e^{V_1}\, H_1.$$

- ? "onshell": $D^2\, \big[\, e^{V_1}\, H_1\, \big] = 4\, \mu\, H_2^\dagger \;\;\Rightarrow\;\;$ wavefunc ren only.

- Removing redundant (d=5) operators by field redefinitions:

$$\begin{aligned}
\mathcal{L}_1 &= \frac{1}{M_*}\int d^2\theta\; \zeta(S)\,(H_2.H_1)^2 + h.c., & [S,T]\\
\mathcal{L}_2 &= \frac{1}{M_*}\int d^4\theta\; \Big\{ a(S,S^\dagger) D^\alpha \Big[b(S,S^\dagger)\, H_2\, e^{-V_1}\Big] D_\alpha \Big[c(S,S^\dagger)\, e^{V_1}\, H_1\Big] + h.c.\Big\}, & [D]
\end{aligned}$$

$$\frac{1}{M_*}\zeta(S) = \zeta_0 + \zeta_1\, m_0\, \theta\theta, \qquad \zeta_0,\ \zeta_1 \sim 1/M_*,$$

$$a(S,S^\dagger) = a_0 + a_1\, S + a_1^*\, S^\dagger + a_2\, S\, S^\dagger, \qquad S = \theta\theta\, m_0,\ \text{spurion}$$

$\Rightarrow$ $\mathcal{L}_2$ removed by general non-linear, field redefinitions in $\mathcal{L}_{MSSM}$

$$\begin{aligned}
H_1 &\to H_1 - \frac{1}{M_*}\overline{D}^2\Big[\delta_1(S,S^\dagger)\; H_2^\dagger\, e^{V_2}\,(i\sigma_2)\Big]^T\\
H_2 &\to H_2 + \frac{1}{M_*}\overline{D}^2\Big[\delta_2(S,S^\dagger)\, H_1^\dagger\; e^{V_1}\,(i\sigma_2)\Big]^T
\end{aligned}$$

$\delta_1 = s_0 + s_1\, S + s_2\, S^\dagger + s_3\, S\, S^\dagger,\qquad \delta_2 = s_0' + s_1'\, S + s_2'\, S^\dagger + s_3'\, S\, S^\dagger,\qquad F$: U^c, D^c, E^c

$\mathcal{L}_2$ removed by suitably chosen s_i, s_i'. $\Rightarrow$ soft terms & μ-term redefinition: $\Rightarrow$ only $\mathcal{L}_1$ left (d=5)